The system matrix can then be estimated as

$$\hat{A}_D = V_2^+ H(1) W_d^+ \tag{28}$$

where the generalized inverses V_2^+ and W_D^+ are computed using the singular-value decomposition of the Hankel matrix $H(0)$ in which

$$V_2 = U_r O_r^{\frac{1}{2}} \qquad W_d = O_r^{\frac{1}{2}} Y_r^T \tag{29}$$

To investigate the effect of sensor placement, the least squares solution in Eq. (28) will be studied as a two-step process. In the first step, assume that the generalized inverse of W_d has been generated and used to postmultiply Eq. (27) to form the expression

$$H(1) W_d^+ = V_2 A_D \tag{30}$$

For fixed actuator locations, sensor placement has no impact upon the generalized inverse of the controllability matrix. Equation (30) represents a straightforward least squares estimation problem in which the discrete system matrix can be estimated as

$$\hat{A}_D = \left(V_2^T V_2\right)^{-1} V_2^T H(1) W_d^+ = (W_0)^{-1} V_2^T H(1) W_d^+ \tag{31}$$

where the Moore–Penrose form of the generalized inverse has been used to invert V_2 to illustrate the effect of sensor placement. For this estimation problem, the observability grammian is the corresponding Fisher information matrix. The best estimate for A_D is obtained by maximizing the determinant of the observability grammian.[6] Therefore, the EfI sensor placement methodology also provides a better estimate of the discrete time system matrix.

Conclusion

In cases where limited numbers of sensors are available for modal identification, it is vital that a systematic method is used for selecting their locations. A method called EfI has been previously devised to place sensors such that the locations maintain the linear independence of the modal partitions. This is required for posttest correlation and finite element model updating. This Note has shown analytically that the EfI method of sensor placement also enhances the modal identification process itself using system realization methods. It was shown that maximizing the independence of the target modes minimizes the size of the observability matrix required to render the system observable. This also minimizes the size of the required block Hankel data matrix used in the modal identification process. By maximizing the determinant of the Fisher information matrix, the EfI method maximizes the observability of the system and maximizes the singular values of the data matrix used in the ERA method to differentiate between target modes and computational modes. It also enhances the estimation of the discrete system matrix. It is believed that the EfI method provides an efficient technique for selecting a sensor configuration that not only provides the modal data required by structural dynamicists, but also produces an enhanced modal identification.

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Sliding Control Using Output Feedback

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Introduction

SLIDING mode control is a powerful robust control technique. It is well known for its design simplicity and robustness to parameter variations. One main drawback of the method is that system states are needed for implementation. There have been many papers on the issue of eliminating this drawback.[1–5] They can be grouped into two categories: static output feedback[1–3] and dynamic output feedback (observers).[4,5] Recently, Wang and Fan[1] proposed a new static output feedback sliding control method that incorporates the initial conditions of the outputs in the sliding variables. To implement their method, one would need to have additional logic for practical application In particular, in the case of regulation or tracking problems, mismatched disturbance could easily knock the trajectory off the desired path, and the control should be able to bring it back. Using Wang and Fan's method would require reinitializing the control after each such occurrence. This could be done by using additional logic. Such an ad hoc fix does not seem desirable.

Thus, it is more appropriate to devise a controller that provides robustness to all initial conditions, parameter variations and mismatched disturbance. The method in the next section can fulfill these requirements.

New Method

Consider a linear system of the form

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + d(t) \tag{1a}$$

$$\dot{x}_2 = (A_{21} + \Delta A_{21})x_1 + (A_{22} + \Delta A_{22})x_2 + B_2 u \tag{1b}$$

$$y = C_1 x_1 + C_2 x_2$$

with $x_1 \in R^{n-m}$, $x_2 \in R^m$, $x = [x_1^T \; x_2^T]^T$, $u \in R^m$, and $y \in R^p$. A_{ij}, B_2, C_1, and C_2 are matrices with appropriate dimensions. Here $d \in R^{n-m}$ denotes mismatched disturbance in the system. Assume the uncertainties ΔA_{21} and ΔA_{22} are all bounded and satisfy the so-called matching conditions, i.e.,

$$\Delta A_{21} = B_2 D_1 \tag{2a}$$

$$\Delta A_{22} = B_2 D_2 \tag{2b}$$

with the D_i unknown but bounded quantities. The sliding variable is defined as

$$S = Gy = GC_1 x_1 + GC_2 x_2 \tag{3}$$

with $S \in R^m$. G can be selected to satisfy Lemma 1 of Wang and Fan,[1] i.e.,

$$\text{rank}[C_2 K - I] \le p - m \tag{4}$$

$$K = (GC_2)^{-1} G \tag{5}$$

with GC_2 of full rank. From Eq. (3), we can express x_2 in terms of S and x_1 to get

$$x_2 = (GC_2)^{-1}(-GC_1 x_1 + S) \tag{6}$$

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Differentiating Eq. (3) and using Eqs. (1) and (6) gives

$$\dot{S} = Mx_1 + NS + GC_1d + GC_2B_2u \tag{7}$$

where

$$M = GC_1A_{11} + GC_2(A_{21} + B_2D_1) - [GC_1A_{12} + GC_2(A_{22} + B_2D_2)](GC_2)^{-1}GC_1$$

$$N = [GC_1A_{12} + GC_2(A_{22} + B_2D_2)](GC_2)^{-1}$$

Since there are uncertainties in M and N, it appears that x_1 is needed to guarantee the sliding condition

$$S^T\dot{S} < 0 \tag{8}$$

However, by utilizing the information of the following reduced-order system, we can get an upper bound on the norm of x_1. Inserting Eq. (6) into Eq. (1a) gives

$$\dot{x}_1 = A_rx_1 + A_{12}(GC_2)^{-1}S + d(t) \tag{9}$$

where

$$A_r = \left(A_{11} - A_{12}(GC_2)^{-1}GC_1\right)$$

The eigenvalues of A_r in Eq. (9) are $\{-\lambda_1, -\lambda_2, \ldots, -\lambda_{(n-m)}\}$ (real part of $\lambda_i > 0$), which can be done by choosing G such that Eq. (3) is satisfied.

Lemma. Consider Eq. (9). Let $\lambda_m > 0$ be the minimum real part of $\{\lambda_1, \lambda_2, \ldots, \lambda_{n-m}\}$. Then we have the following.

1) The $\|\exp(A_rt)\| \leq \gamma\exp(-\lambda_mt)$ for some $\gamma > 0$.

2) Also, $\|x_1(t)\|$ is bounded by $w(t)$ after a finite period of time with $w(t)$ the solution of

$$\dot{w}(t) = -\lambda_w w(t) + \gamma\left[\left\|A_{12}(GC_2)^{-1}\right\|\left\|S\right\| + \bar{d}\right]$$

$$\|d(t)\| < \bar{d}, \qquad \lambda_w < \lambda_m, \qquad w(0) > 0 \tag{10}$$

Proof: It is obvious that 1 holds for some positive constant γ. To see 2, we solve Eq. (9) to yield

$$x_1(t) = e^{A_rt}x_1(0) + \int_0^t e^{A_r(t-\tau)}\left[A_{12}(GC_2)^{-1}S + d(t)\right]\mathrm{d}\tau$$

Hence,

$$\|x_1(t)\| \leq \gamma e^{-\lambda_mt}\|x_1(0)\| + \int_0^t \gamma e^{-\lambda_m(t-\tau)} \times \left[\left\|A_{12}(GC_2)^{-1}\right\|\left\|S\right\| + \left\|d(t)\right\|\right]\mathrm{d}\tau \tag{11}$$

by using 1. The solution of Eq. (10) is given by

$$w(t) = e^{-\lambda_wt}w(0) + \int_0^t \gamma e^{-\lambda_w(t-\tau)}\left[\left\|A_{12}(GC_2)^{-1}\right\|\left\|S\right\| + \bar{d}\right]\mathrm{d}\tau \qquad w(0) > 0 \tag{12}$$

Comparing the right-hand sides of Eqs. (11) and (12) and noting the fact that $\lambda_w < \lambda_m$, we see that

$$w(t) \geq \|x_1(t)\|$$

after a finite period of time.

Returning to Eq. (7) and expressing it in the form of

$$\begin{bmatrix} \dot{S}_1 \\ \cdots \\ \dot{S}_m \end{bmatrix} = \begin{bmatrix} M_1 \\ \cdots \\ M_m \end{bmatrix} x_1 + \begin{bmatrix} N_1 \\ \cdots \\ N_m \end{bmatrix} S + GC_1d(t) + GC_2B_2u \tag{13}$$

We propose the following sliding controller:

$$u = (GC_2B_2)^{-1}\{-\mathrm{diag}\{H_1, \ldots, H_m\}w(t) - \mathrm{diag}\{L_1, \ldots, L_m\}\|S\| - \mathrm{diag}\{K, \ldots, K\}\bar{d}\}\mathrm{sgn}(S) \tag{14}$$

where

$$H_i \geq \|M_i\|, \qquad L_i \geq \|N_i\|, \qquad i = 1, 2, \ldots, m$$

$$K \geq \|GC_1\|, \qquad \mathrm{sgn}(S) = [\mathrm{sgn}(S_1), \ldots, \mathrm{sgn}(S_m)]^T$$

Substituting Eq. (14) into Eq. (13) and using the fact that $w(t) \geq \|x_1(t)\|$ from the lemma, one can see that after a finite period of time

$$S_i\dot{S}_i < 0 \tag{15}$$

Hence, S tends to zero globally.

Simulations

The aircraft model[1] is used here. Comparing with the form described in Eq. (1), we get the following system matrices:

$$A_{11} = \begin{bmatrix} -0.277 & 1 \\ -17.1 & -0.178 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} -0.0002 \\ -12.2 \end{bmatrix}$$

$$A_{21} = [0 \quad 0], \qquad A_{22} = [-6.67], \qquad B_2 = [6.67]$$

$$C_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Let $x_1 = [\alpha \;\; q]^T$ and $x_2 = \delta_e$. The uncertainties in $[D_1 \;\; D_2]$ in Eq. (2) are assumed to be

$$0.1[\sin(t) + \cos(2t) \quad \sin(3t)\cos(t) \quad -1 + \sin(2t)\cos(3t)]$$

The mismatched disturbance $d(t)$ is [0 0.5] after $t > 2$ s. G is chosen to place the eigenvalues of the reduced-order system at

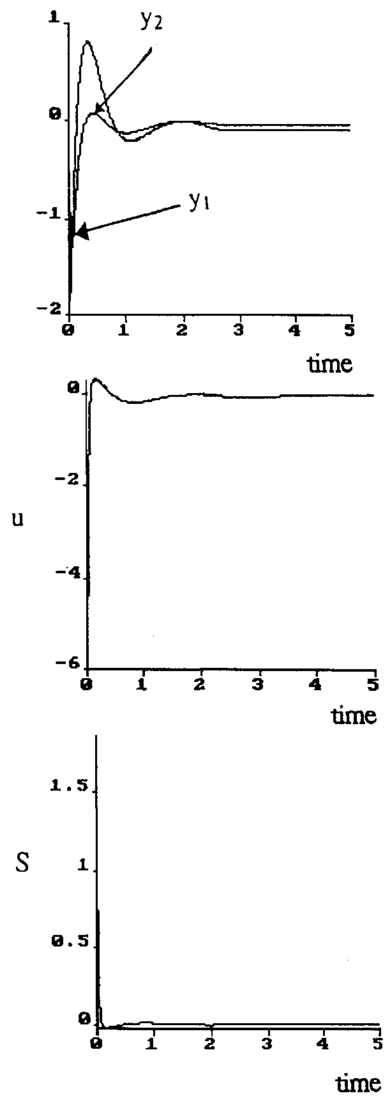

Fig. 1 Performance of Wang and Fan's method.

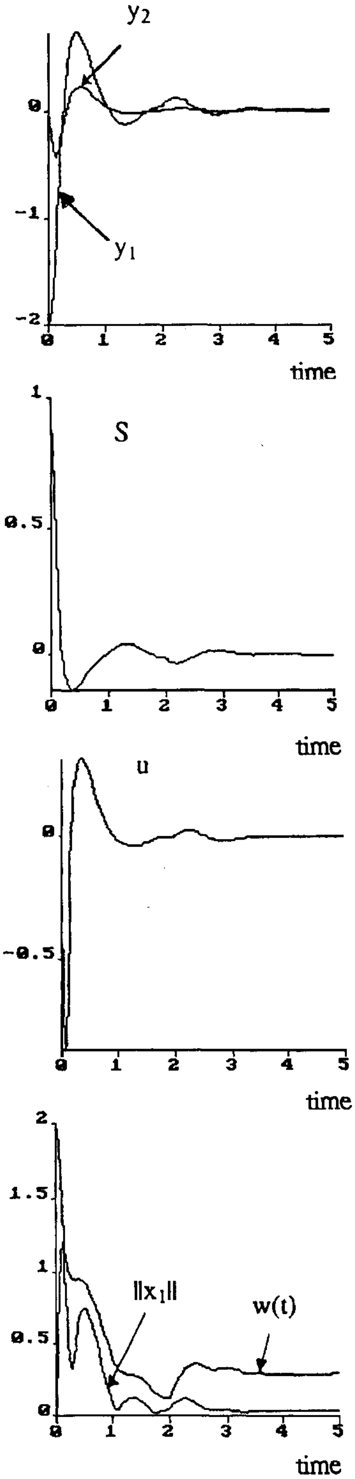

Fig. 2 Performance of our controller.

$-3.06 \pm j3.06$. G is given by $[-0.4635 \;\; 1]$. It can be verified that M and N in Eq. (13) satisfy

$$\|M\| < 10, \qquad \|N\| < 2.5$$

The value of K is 0.44. Hence, we arrive at the following controller:

$$u = -(10w(t) + 2.5|S| + 0.44\bar{d})\mathrm{sgn}(s)/6.67 \qquad (16)$$

with $\mathrm{sgn}(S) = S/(|S| + 0.5)$ to reduce control chattering. The λ_w in Eq. (10) is chosen to be 3 and the γ in Eq. (10) is set as 1.5. We let $w(0) = 0.01$ since we do not know the initial conditions of y_i. Figure 1 shows the performance of Wang and Fan.[1] Figure 2 shows the performance of our method. It can also be seen that the $\|x_1\|$ is smaller than $w(t)$ after approximately 0.1 s. It is clear that our method is quite good in terms of smaller error and control effort.

Conclusions

We proposed a new sliding control method using output feedback that can guarantee global stability. The current scheme is very simple in structure because no observer is needed. The computational requirement is also very small. It is robust to all initial conditions, mismatched disturbance and parametric uncertainties.

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Parametric Uncertainty Reduction in Robust Missile Autopilot Design

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I. Introduction

THE μ-synthesis design procedure provides a very general framework for the design of robust autopilots.[1–5] The μ-synthesis technique[6] seeks to minimize the structured singular value[7] of the closed-loop transfer function from disturbances and perturbations to reference outputs and the outputs to the perturbations. Optimization is accomplished by alternatively minimizing the infinity norm of the closed-loop transfer function over the set of stabilizing controllers[8] and minimizing the maximum singular value of this transfer function over the set of D-scales.[7] The frequency-dependent D-scales are approximated by rational transfer functions and appended to the plant between iterations.

The μ-synthesis algorithm is not guaranteed to converge to the global minimum, and if it does converge to the global minimum, this minimum is only guaranteed to be optimal in terms of robust performance for two or fewer perturbations.[6,7] Computer-aided design software may, therefore, fail to find a controller that yields robust performance, even when such a controller exists. It is reasonable to expect the μ-synthesis algorithm to perform better if the number of perturbations can be reduced. This hypothesis has

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